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CHARACTERIZATION OF THE RANDOM ARRAY PEAK SIDELOBE USING LEVEL --ETC(U)
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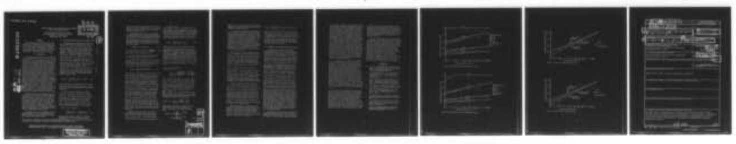

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CHARACTERIZATION OF THE RANDOM ARRAY PEAK SIDELOBE USING LEVEL CROSSING THEORY

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Abstract

An upper bound on the height of the peak sidelobe of the random array is derived using the theory of level crossings of a random process. This bound is obtained by calculating the expected number of upcrossings of a given level by the array power pattern. The bound is compared with computer simulation results.

I. INTRODUCTION

The linear random array has been studied by several authors, e.g. [1]-[4]. In this type of array all of the antenna elements lie on a line, and their positions are chosen randomly and independently from some probability distribution. So any array with specified element locations designed in this way is a particular realization from the set of possible arrays determined by the given probability distribution. By choosing positions in this way, fewer elements may be used than would be necessary in an array with uniformly spaced elements, and grating lobes, which are sidelobes of height equal to the height of the main lobe, will be eliminated [1]. Because of this property a random array could be useful in an application in which a few elements must be spread over a large aperture. Since the element locations are chosen randomly, the power pattern of the array is a sample function of a random process which is a function of angle. It is important to be able to characterize the height of the peak (i.e., highest) sidelobe of the power pattern. A complete characterization of the height of the peak sidelobe would be given by its probability density function (pdf).

Approximations to this probability density function have been derived [1]-[4] by considering samples of the power pattern at a number of points. Different results will be obtained in this paper, where an upper bound on the height of the peak sidelobe will be derived by using the theory of level crossings of a random process (discussed, for example, in [5]-[7]). Since it is important in antenna array design to make the sidelobes as low as possible, a probabilistic upper bound on the peak sidelobe height can be of considerable significance.

This upper bound will be compared with computer simulations. It will be shown that the derived result yields a reasonably tight upper bound on the peak sidelobe height.

II. SIDELOBE CHARACTERISTICS OF THE RANDOM ARRAY

Consider a linear antenna array with N elements distributed over an aperture of length L. Let $y_1, y_2, \ldots, y_N$ be the randomly chosen locations of the array elements with respect to an origin arbitrarily picked at the midpoint of the aperture. Suppose the wavelength of incident radiation is λ, for which the wave number is $k=2\pi/\lambda$. If the power received at each element is the same, then $S_i(\theta) = \exp(jky_i \sin\theta)$ is the complex amplitude of the signal received at the i^{th} element from a sinusoidal point source at angle θ from array broadside. The complex weight needed to bring the i^{th} element signal into phase with the signal at the origin, so that the signals may be coherently summed, is $W_i(\theta) = \exp(-jky_i \sin\theta)$. The complex beam pattern $B(\theta_0,\theta)$ is defined to be the complex amplitude of the signal received by the array from a sinusoidal source in the direction θ with element weights fixed at $W_i(\theta_0)$. Thus

$$B(\theta_0,\theta) = \sum_{i=1}^{N} \exp[jky_i(\sin\theta - \sin\theta_0)]. \qquad (1)$$

The power pattern $|B(\theta_0,\theta)|^2$ of the array is the power received from a sinusoidal source at angle θ when the array is weighted to look in the direction θ_0.

Since the element locations are random quantities, $B(\theta_0,\theta)$ and $|B(\theta_0,\theta)|^2$ are random processes. The special case in which $\theta_0=0$ will be considered, although an extension is possible to the case in which $\theta_0 \neq 0$. Let $u=\sin\theta$, $X(u) = B(0,\theta)$, and let $X_1(u)$ and $X_2(u)$ be the real and imaginary components of $X(u)$. Note that u lies in the interval [-1,1]. From (1),

$$X(u) = X_1(u) + jX_2(u)$$
$$= \sum_{i=1}^{N} \cos(ky_i u) + j \sum_{i=1}^{N} \sin(ky_i u), \qquad (2)$$

and the power pattern is $|X(u)|^2$. Since the beam pattern X(u) has the symmetry property

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$X(-u) = X^*(u)$, $X(u)$ in the interval $[0,1]$ determines $X(u)$ in the interval $[-1,0)$. So it is sufficient to consider the beam pattern $X(u)$ only over the interval $[0,1]$.

The random process $X(u)$ can be partially described in terms of the means, the autocovariance functions, and the crosscovariance function of $X_1(u)$ and $X_2(u)$. The special case in which the element positions are independent and uniformly distributed over the interval $[-L/2, L/2]$ will be considered. The mean value of the process $X_1(u)$ is [4]

$$E\{X_1(u)\} = N \operatorname{sinc}(uL/\lambda) \tag{3}$$

for $u \in [0,1]$, where $\operatorname{sinc}(uL/\lambda) = \dfrac{\sin(\pi uL/\lambda)}{\pi uL/\lambda}$. The mean value of $X_2(u)$ is $E\{X_2(u)\} = 0$.

It can be shown that the crosscovariance function $K_{X_1X_2}(u_1,u_2)$ is zero for all $u_1, u_2 \in [0,1]$. In the sidelobe region of the beam pattern, where u is much larger than λ/L, the autocovariance functions of $X_1(u)$ and $X_2(u)$ are approximately equal and are given by

$$K_{X_1}(v) \doteq K_{X_2}(v) \doteq \frac{N}{2} \operatorname{sinc}(vL/\lambda), \tag{4}$$

where $v = u_1 - u_2$. The mean of the process $X_1(u)$ can be neglected in the sidelobe region if it is small in comparison with the standard deviation $\sqrt{N/2}$ of the processes. The mean of $X_1(u)$ will first be neglected, and later its effect on the peak sidelobe will be considered. Therefore, here it will be assumed that the processes $X_1(u)$ and $X_2(u)$ are wide-sense stationary in the sidelobe region of the beam pattern. The processes $X_1(u)$ and $X_2(u)$, for a given u, are each equal to a sum of N independent, identically distributed random variables. A multivariate form of the central limit theorem [8] can be used to show that, in the sidelobe region, $X_1(u)$ and $X_2(u)$ can be approximated to be normal random processes with zero means and autocovariance functions given by (4); henceforth they will be treated as if they were normal processes. Since $X_1(u)$ and $X_2(u)$ are assumed to be normal and are uncorrelated, they must be independent, and since each is wide-sense stationary, they must both be stationary.

The peak sidelobe height is the highest local maximum, other than the maximum at $u=0$ which is the main lobe, of the power pattern $|X(u)|^2 = X_1^2(u) + X_2^2(u)$. The expected number of upcrossings of a level z by the square root $R(u) = \sqrt{X_1^2(u) + X_2^2(u)}$ of the power pattern, in any interval, can now be calculated. This will yield a bound on the probability of $R(u)$ exceeding z in the sidelobe region. It can be shown that the expected number $E\{U_z\}$ of upcrossings in an interval of length ℓ of the level z by any stationary random process $A(u)$ is given by [7]

$$E\{U_z\} = \ell \int_0^\infty |a'|\, f_{AA'}(z,a')\,da' \tag{5}$$

where $f_{AA'}(a,a')$ is the joint pdf of the process $A(u)$ and its derivative $A'(u)$. (It will be assumed here that $A(u)$ has continuous sample functions. The power pattern of the random array has this property). This expectation value has been calculated for the case in which $A(u)$ is normal [5] and the case in which $A(u)$ is the envelope of a normal process [6]. Methods used by Rice [5] will be used here to determine $E\{U_z\}$ for $R(u)$. The joint pdf of $X_1(u)$ and $X_2(u)$, and their derivatives $X_1'(u)$ and $X_2'(u)$, at any point u, is

$$f_{X_1,X_2,X_1',X_2'}(x_1,x_2,x_1',x_2') =$$

$$\frac{1}{(2\pi)^2 \lambda_0 \lambda_0} \exp\left[\frac{-1}{2}\left\{\frac{x_1^2+x_2^2}{\lambda_0} + \frac{x_1'^2+x_2'^2}{\lambda_2}\right\}\right] \tag{6}$$

where $\lambda_0 = N/2$ is the variance of $X_1(u)$ and $X_2(u)$, and $\lambda_2 = (N/6)(\pi L/\lambda)^2$ is the variance of $X_1'(u)$ and $X_2'(u)$. These variances can be calculated using the technique described in [6], for example. By using the change of variables $x_1 = r\cos\theta$, $x_2 = r\sin\theta$, the joint pdf of $R(u) = \sqrt{X_1^2(u) + X_2^2(u)}$, $\theta(u) = \tan^{-1}[X_1(u)/X_2(u)]$, and their derivatives $R'(u)$ and $\theta'(u)$, can be found. Integrating over all possible values of θ and θ' yields the joint pdf of $R(u)$ and $R'(u)$:

$$f_{RR'}(r,r') = \frac{r}{\sqrt{2\pi\lambda_2}\,\lambda_0} \exp\left(\frac{-r^2}{2\lambda_0} - \frac{r'^2}{2\lambda_2}\right) \tag{7}$$

Using equation (5) for the expected number of upcrossings of z by $R(u)$, and setting $\ell = (1-\lambda/L)$, the length of the sidelobe region, gives

$$E\{U_z\} = (1-\lambda/L)\sqrt{\frac{\lambda_2}{2\pi}}\,\frac{z}{\lambda_0}\,e^{-z^2/2\lambda_0}$$

$$= (1-\lambda/L)\sqrt{\frac{\pi}{3}}\,\frac{L}{\lambda}\,\frac{z}{\sqrt{N}}\,e^{-z^2/N} \tag{8}$$

Equation (8) is the main result of this section, and will be used to set an upper bound on the height of the peak sidelobe.

To use (8) to study the peak sidelobe of R(u), the relationship between level crossings of a random process and local maxima of that process must be considered. There are at least as many local maxima above z as there are upcrossings of z; thus the probability of an upcrossing of z is equal to the probability of having at least one local maximum above z. Following methods used by Rice [5], it can be shown that the expected number of local maxima above z approaches the expected number of upcrossings of z for high levels of z. This justifies the use of level crossings to study the local maxima of R(u).

For the case of R(u) the expected value $E\{U_z\}$ of the number of upcrossings in the sidelobe region is an upper bound on the probability $P\{U_z\}$ of at least one upcrossing of z in that region. The number of upcrossings of a level z can assume only non-negative integer values. Letting $p_i(z)$ be the probability of exactly i upcrossings of z, it is clear that $E\{U_z\} = \sum_{i=0}^{\infty} i p_i(z)$, and $P\{U_z\} = \sum_{i=1}^{\infty} p_i(z)$. Therefore $P\{U_z\} \leq E\{U_z\}$, and $E\{U_z\}$ is an upper bound on $P\{U_z\}$. It is reasonable to expect that this bound becomes tight for small values of $E\{U_z\}$, since in this case the probability of obtaining more than one upcrossing in the sidelobe region is negligible compared to the probability of obtaining zero or one upcrossing.

The case in which the mean $E\{X_1(u)\}$ of $X_1(u)$ can be neglected has been considered. Since $E\{X_1(u)\} = N \operatorname{sinc}(uL/\lambda)$ and the standard deviation of $X_1(u)$ is approximately $\sqrt{N/2}$, we see that as N increases while L/λ remains constant, the mean will increase relative to the standard deviation, and will make a large contribution to the beam pattern. The case in which $E\{X_1(u)\}$ cannot be neglected will now be treated. The process $X_1(u)$ is no longer stationary, and the expected number of upcrossings of z by R(u) is not given by (5).

The expected number of upcrossings by an arbitrary, nonstationary random process can be calculated by integrating over the interval in question. Specifically the expected number $E\{U_z\}$ of upcrossings in the interval $[\lambda/L,1]$ of the level z by the square root R(u) of the power pattern is given by ([6], Chapter 13)

$$E\{U_z\} = \int_{\lambda/L}^{1} du \int_{0}^{\infty} |r'(u)| f_{RR'}(z,r'(u);u)dr', \quad (9)$$

where $f_{RR'}(r(u),r'(u);u)$ is the joint probability density function of R(u) and R'(u), written as a function of $u\varepsilon[\lambda/L,1]$. (Note that u is not a random variable). The expected value $E\{U_z\}$ is not obtainable in closed form, and must be calculated by numerical integration. Computed results based on (9) will be presented in graphical form in the next section.

III. COMPARISON WITH COMPUTER SIMULATION RESULTS

The bounds on the peak sidelobe height given by equations (8) and (9) are presented graphically in Figures 1 and 2, where the solid lines represent the bound given by (9) and the dashed lines those given by (8). The value of the normalized level z^2/N^2 versus the normalized length L/λ of the array is plotted for different numbers of elements N, and the indicated values of $E\{U_z\}$. The normalized level z^2/N^2 is used because from it is obtained the height of the peak sidelobe of the power pattern relative to the height of the main beam. In Figure 1, the value of $E\{U_z\}$ is 0.1, and in Figure 2, $E\{U_z\}$ is 0.3. From Section II, we know that the probability $P\{U_z\}$ of at least one upcrossing of z is less than or equal to $E\{U_z\}$. Thus for a given N and L/λ, the probability $P\{U_z\}$ of the normalized height of the peak sidelobe of the power pattern exceeding the value z^2/N^2 given by the curve of Figure 1 or 2 is less than or equal to $E\{U_z\}$. This is equivalent to saying that the normalized height exceeded by the peak sidelobe with probability $E\{U_z\}$ is less than or equal to the normalized height z^2/N^2 given by the curve of Figure 1 or 2. It can be seen that there are differences between the calculations based on the assumption that the beam pattern is stationary with zero mean and the more precise calculation which considers the nonconstant mean of the beam pattern. The peak sidelobe upper bound increases due to the mean term, particularly where the density $N/(L/\lambda)$ of array elements per array length is high. It is reasonable to expect this difference to increase when N increases or L/λ decreases, since in these cases the mean term becomes larger with respect to the standard deviation of the beam pattern over the sidelobe region.

Figures 1 and 2 also contain the results of computer simulations that were done to study the validity of the upper bounds. For given values of N and L/λ, a set of random arrays was formed

by generating independent random numbers to represent element positions uniformly distributed over $[-L/2, L/2]$. The power pattern of each array was calculated, and the peak sidelobe was found. For each value of N and L/λ, an empirical cumulative distribution function of the peak sidelobe value was formed from this set of arrays. In Figure 1, the points represent the normalized level which exceeded the peak sidelobe height of 90% of the arrays, and in Figure 2, the points represent the level which exceeded the peak sidelobe height of 70% of the arrays. For each value of N and L/λ shown, 100 arrays were generated, except for $N=20$, $L/\lambda=200$, for which 50 arrays were generated. However, for each N and L/λ, the points in Figures 1 and 2 are taken from the same empirical distribution function, and do not represent the results of independent experiments. It can be seen that in all cases the simulations fell below the curve given by the more precise upper bound obtained from (9), and for lower densities of elements $N/(L/\lambda)$, the experimental points fell below the upper bound calculated from (8).

It is likely that the density of elements will not be high for practical random array designs. For an antenna array with a density of $N/(L/\lambda)=2$ or greater, a periodic array can be designed with no grating lobes and low sidelobes, and for arrays with such densities, mutual coupling between elements changes the beam pattern, making equation (1) invalid. Using random element positions would be most useful in applications where a low density of elements must be used, since in that case the effect of mutual coupling is small, and the use of equally spaced elements could lead to a large number of grating lobes. In these cases, the upper bound given by (8) could be used to determine the peak sidelobe height. This is fortunate since (8) is simple to compute. Using (9) to determine the upper bound would only be necessary in the less practical case of a high element density.

In Section II, the approximate covariance function (4) was derived based on the assumption that the region near the main beam was excluded from the sidelobe region. The region $[\lambda/L, 1]$ has been considered to be the sidelobe region for the calculations and simulations presented in this section, but the results might have been more valid if a smaller sidelobe region, such as $[3\lambda/L, 1]$, had been used. The region $[\lambda/L, 1]$ was studied because for practical array design, it is necessary to determine the peak sidelobe over the entire region, not only over the region in which the theory yields a very accurate result. Since the region near the main beam exhibits some of the properties of deterministic arrays [4], an alternative method of studying the peak sidelobe, which will not be considered here, might be to use the theory to study the restricted region in which the approximations are very accurate, and treat the region near the main beam as if it were deterministic.

Figures 3 and 4 compare the results derived in this paper with previous results concerning the peak sidelobe of the random array. Steinberg [3], [4] used a technique in which the beam pattern was sampled at a number of points, an estimate of the peak sidelobe height was formed, and a correction term based on interpolation was added to the estimate. The solid and dashed lines and simulation points in Figures 3 and 4 were taken from Figures 1 and 2, respectively, and the dotted line was calculated using Steinberg's technique. It can be seen that Steinberg's estimate is equal to or greater than the upper bound derived in this paper, and lies above the simulations reported in this paper, for the parameter values that were considered.

REFERENCES

1. Y. T. Lo, "A Mathematical Theory of Antenna Arrays with Randomly Spaced Elements," IRE Trans. Antennas and Propagation, Vol. AP-12, pp. 257-268, May 1964.
2. V. D. Agrawal and Y. T. Lo, "Distribution of Sidelobe Level in Random Arrays," Proceedings IEEE, Vol. 57, pp. 1764-1765, October 1969.
3. B. D. Steinberg, "The Peak Sidelobe of the Phased Array Having Randomly Located Elements," IEEE Trans. Antennas and Propagation, Vol. AP-20, pp. 129-136, March 1972.
4. B. D. Steinberg, Principles of Aperture and Array System Design, New York: Wiley, 1976.
5. S. O. Rice, "The Mathematical Analysis of Random Noise," Bell System Tech. J., Vol. 23, pp. 282-332, 1944, and Vol. 24, pp. 46-156, 1945.
6. H. Cramer and M. R. Leadbetter, Stationary and Related Stochastic Processes, New York: Wiley, 1967.
7. I. F. Blake and W. C. Lindsey, "Level Crossing Problems for Random Processes," IEEE Trans. Information Theory, Vol. IT-19, pp. 295-315, May 1973.
8. W. Feller, An Introduction to Probability Theory and its Applications, Volume II, New York: Wiley, 1966.

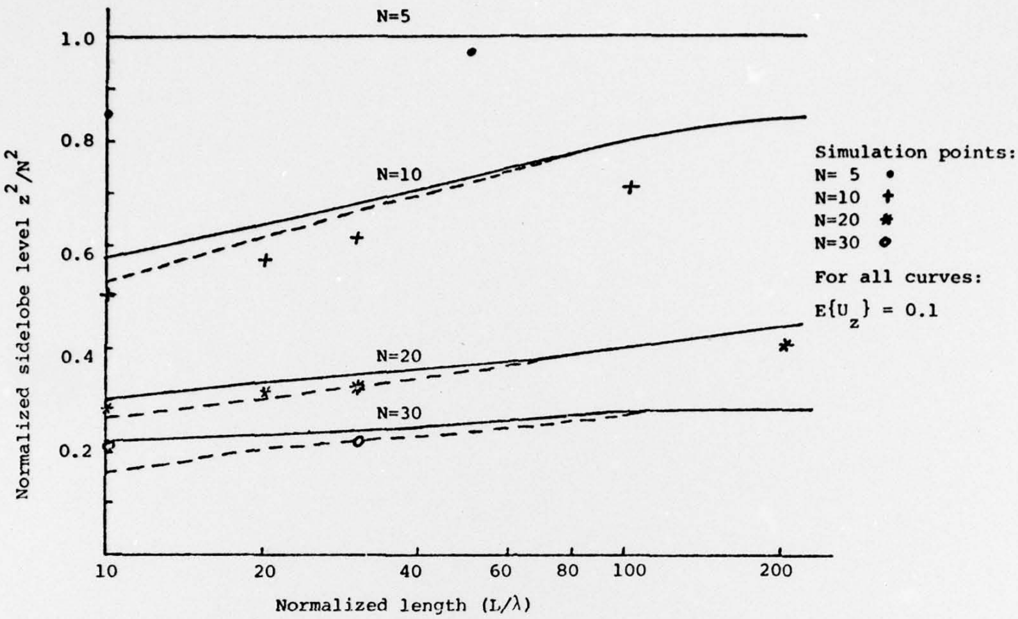

Figure 1. Upper bound curves and simulation results.

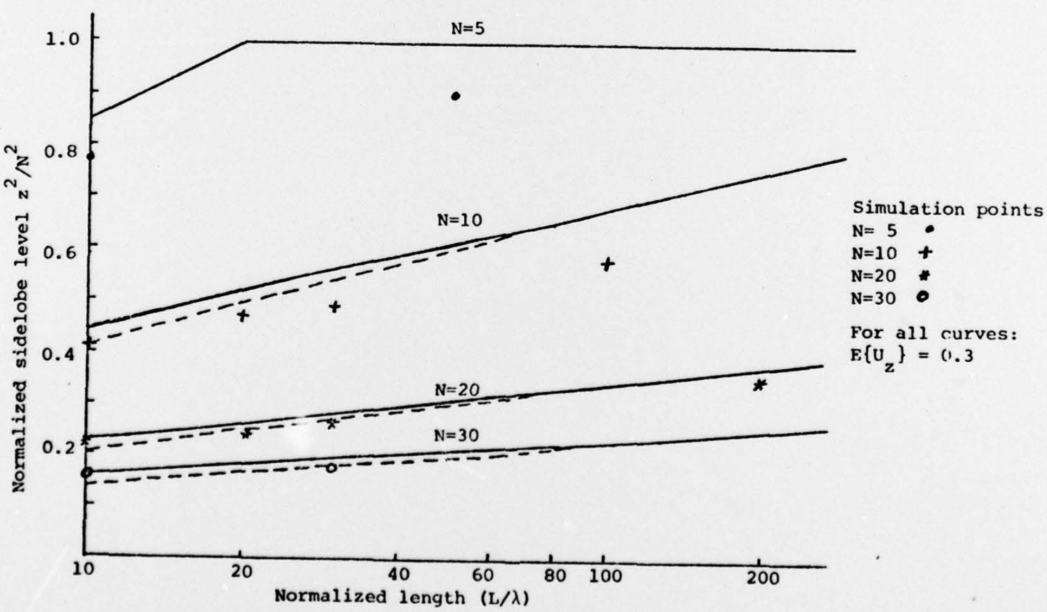

Figure 2. Upper bound curves and simulation results.

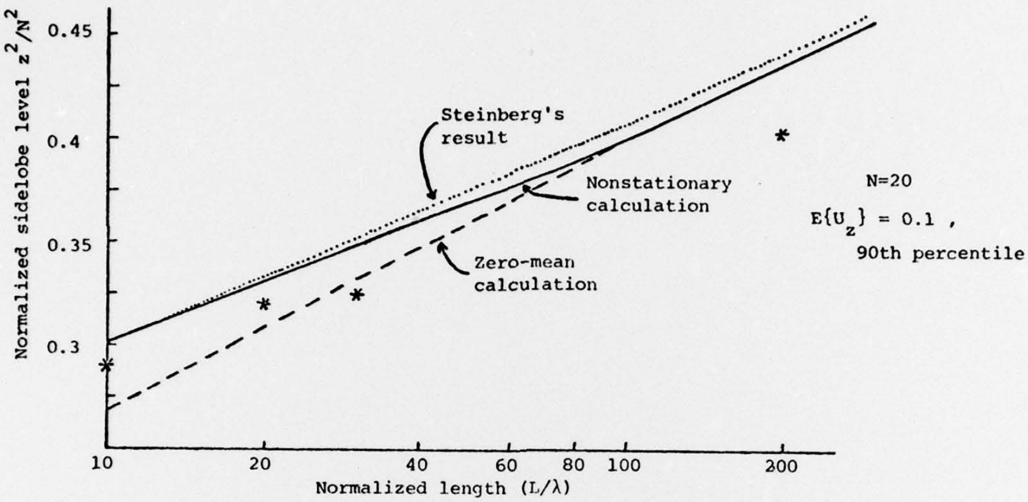

Figure 3. Comparison with previous results.

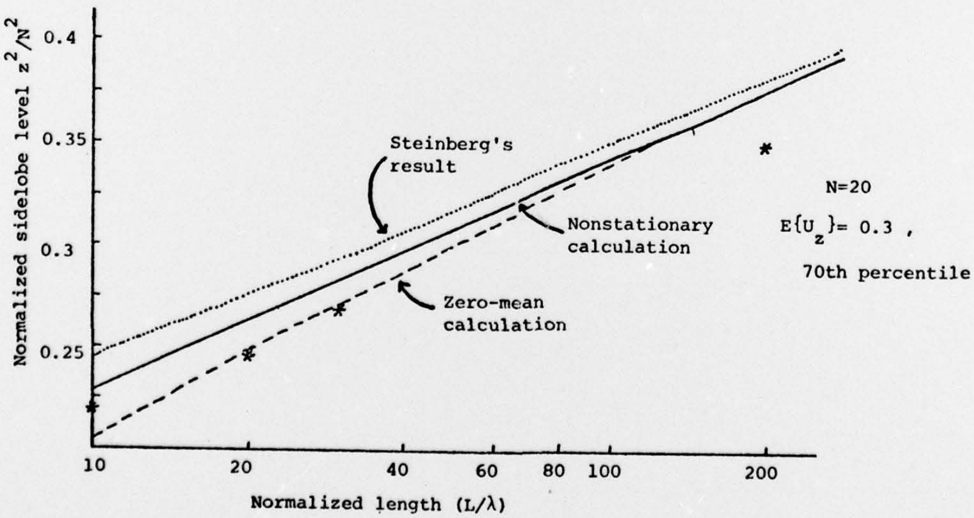

Figure 4. Comparison with previous results.

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